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A NONLINEAR STUDY OF AI VELORUM

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ABSTRACT

Using results from earlier studies of double-mode dwarf Cepheids by Cox, King, and Hodson, hydrodynamic calculations have been undertaken for AI Vel. The earlier derived evolution theory mass of 1.8 $M_{\odot}$, a luminosity of 23 $L_{\odot}$ and a T_e of 7500 K give the observed period of 0.11 day and the observed period ratio $\Pi_1/\Pi_0 = 0.773$. A cooler T_e than the observed one (7620 K) was used because 7620 K is beyond the fundamental pulsation mode blue edge. The composition used is X = 0.70 Z = 0.01. This lower than normal Z is necessary to make the period ratio as large as observed. The goal is to see if double-mode behavior, due to either mode switching or a permanent state, can be predicted for the model. Progress in converging the model to a periodic pulsation solution by the von Sengbusch-Stellingwerf relaxation method will be reported.

A NONLINEAR STUDY OF AI VELORUM

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IN EARLIER STUDIES OF DOUBLE-MODE DWARF CEPHEIDS BY COX, KING, AND HODSON IT WAS PROPOSED THAT THE OBSERVED PERIOD RATIOS OF THE TWO SIMULTANEOUS PULSATION MODES COULD BE CORRECTLY PREDICTED THEORETICALLY FOR STARS OF MASS 1.1 TO 2.2 $M_{\odot}$ IN THEIR NORMAL GIANT EVOLUTION. PREVIOUSLY, SOME AUTHORS SUCH AS BESSELL, PETERSEN AND JORGENSEN, JONES, DZIEMBOWSKI, AND KOZLOWSKI, AND SIMON CONCLUDED THAT THE DWARF CEPHEIDS HAD LOW MASSES PROBABLY IN A POST-RED GIANT EVOLUTIONARY STAGE. LATER BREGER, BESSELL, AND MCNAMARA AND LANGFORD FELT THAT THE DWARF CEPHEIDS OR AI VELORUM STARS WERE MERELY LARGE AMPLITUDE δ SCUTI VARIABLES. MANY OBSERVED QUANTITIES, SUCH AS PERIOD DISTRIBUTIONS, WESSELINK RADII, SPACE MOTIONS, METAL ABUNDANCES, GRAVITIES, ETC. SHOW THIS. THE REMAINING PROBLEM WAS THAT THE PERIOD RATIOS WERE BEST EXPLAINED BY THE LOW MASS POPULATION II STARS. OUR WORK AND THAT BY STELLINGWERF SHOWED THAT THE LARGE OBSERVED PERIOD RATIOS OF 0.763 TO 0.778 COULD BE OBTAINED EVEN WITH THE NORMAL HEAVY ELEMENT COMPOSITION Z AS 0.02. FOR AI VEL, THE PROTOTYPE VARIABLE AT 0.11 DAY, THE BEST LINEAR THEORY PERIOD RATIO FIT OF 0.773 BY COX, KING, AND HODSON IS 1.8 $M_{\odot}$, 23 $L_{\odot}$, 7500 K AND A COMPOSITION $X = 0.70$ $Z = 0.01$ WHICH PARAMETERS ARE CONSISTENT WITH BOTH NORMAL POST-MAIN SEQUENCE EVOLUTION AND LINEAR PULSATION THEORIES.

TABLE 1 SHOWS ALL THE KNOWN DOUBLE-MODE DWARF CEPHEIDS INCLUDING OUR STAR AI VEL, FOR THIS NONLINEAR STUDY. AS WE WILL

SEE LATER, THE 7620 K VALUE FOR T_E IS EXACTLY AT THE FUNDAMENTAL RADIAL MODE BLUE EDGE. TO MAKE OUR STUDY WE HAVE USED T_E AS 7500 K TO GET A NON-ZERO FUNDAMENTAL MODE GROWTH RATE. THE PAPER WITH THESE DATA WILL BE IN THE MARCH 15 ASTROPHYSICAL JOURNAL.

ALSO INCLUDED IN THIS AP. J. PAPER IS A TABLE OF PERIOD RATIO FOR MASSES AND COMPOSITIONS OF INTEREST. THIS IS ON TABLE 2. THE PERIOD RATIOS ARE OBTAINED FOR A CONSTANT PERIOD OF 0.11 DAY WHICH IS A LINE OF ALMOST CONSTANT RADIUS FOR EACH MASS IN THE HERTZSPRUNG RUSSELL DIAGRAM. THIS LINE OF CONSTANT PERIOD IS ALSO A LINE OF CONSTANT PERIOD RATIO, AND IT IS GIVEN FOR T_E VALUES ALL ACROSS THE INSTABILITY STRIP BETWEEN 7000 K AND 8000 K.

THE COMPOSITIONS THAT GIVE THE OBSERVED PERIOD RATIO OF 0.773 ARE THE CARSON C312 MIXTURE, DEUPREE IV, DEUPREE V, AND COGAN II. DEPLETION OF THE HELIUM CONTENT BY GRAVITATIONAL SETTLING IN THESE HIGH GRAVITY SLOWLY EVOLVING STARS ALSO CAN GIVE THE OBSERVED PERIOD RATIO. EXCEPT FOR THE CONTROVERSIAL CARSON C312 MIXTURE, A VALUE OF $Z = 0.01$ SEEMS REQUIRED; THE COX-DAVIS VI MIXTURE HAS THE PERIOD RATIO TOO LOW. THUS IN OUR 50 ZONE MODEL FOR NONLINEAR STUDIES WE HAVE USED THE DEUPREE IV COMPOSITION WITH NO SURFACE HELIUM DEPLETION.

NOW THE IDEA IS SIMPLE. WE WANT TO DEMONSTRATE THE DOUBLE-MODE BEHAVIOR BY PERIODIC NONLINEAR CALCULATIONS. LINEAR THEORY ANALYSIS OF THE NONLINEAR PERIODIC SOLUTIONS SHOULD SHOW WHETHER A GIVEN FULL AMPLITUDE MODE IS STABLE OR UNSTABLE AGAINST DECAY TO ANOTHER. AS STELLINGWERF HAS DISCOVERED FOR RR LYRAE VARIABLES AND SOME 1.6 $M_\odot$ MODELS, THERE MIGHT BE A REGION IN THE H-R DIAGRAM

WHERE THE FUNDAMENTAL AND FIRST OVERTONE MODES WANT TO DECAY TO EACH OTHER. THAT WOULD GIVE A MIXED MODE PULSATION. HOWEVER, AS I BELIEVE YOU ALL KNOW, THE PERMANENT DOUBLE-MODE BEHAVIOR (CHANGING ONLY ON AN EVOLUTION TIMESCALE) FOR THE 1.6 $M_{\odot}$ CASE COULD NOT BE CONFIRMED BY HODSON AND COX. FURTHER, THE RR LYRAE PERMANENT DOUBLE-MODE DOMAIN IS CLEARLY BEYOND THE RED EDGE. WE EXPECT AS OF NOW, THEREFORE, THAT ONLY MODE SWITCHING FROM F TO 1H OR VICE VERSA COULD BE THE CAUSE OF THE TWO SIMULTANEOUS RADIAL PULSATION MODES FOR ANY DOUBLE-MODE VARIABLE SUCH AS OUR DWARF CEPHEID AI VEL.

THE PROBLEM WITH MODE SWITCHING IN THE CASE OF THE DOUBLE-MODE CEPHEIDS IS THAT THE SWITCHING TIME IS OF THE ORDER OF MAGNITUDE OF 10^2 YEARS WHILE THE EVOLUTION TIME AS PULSATORS IS 10^6 YEARS. THUS ONLY 10^{-4} OF THE CEPHEIDS SHOULD EXHIBIT DOUBLE-MODE BEHAVIOR, WHEREAS ABOUT 25 PERCENT ACTUALLY DO. A DOMAIN OF INSTABILITY TOWARD EACH MODE ONE QUARTER THE WIDTH OF THE INSTABILITY STRIP IS INDICATED, BUT NO DOMAIN AT ALL HAS YET TO BE FOUND BY NONLINEAR PERIODIC CALCULATIONS. THE LATEST DEEP HELIUM ENRICHMENT MODELS FOR THE DOUBLE-MODE CLASSICAL CEPHEIDS MAY GIVE SUCH A DOMAIN BUT THE CALCULATIONS HAVE NOT YET BEEN DONE. CERTAINLY THE RESONANCE IDEAS OF SIMON, IF CORRECT, ALSO GIVE VERY NARROW DOUBLE MODE REGIONS IN THE INSTABILITY STRIP.

TO ILLUSTRATE THIS SITUATION, LET ME SHOW THE BEST AVAILABLE STORY FOR RR LYRAE VARIABLES. FIGURE 1 GIVES STELLINGWERF'S GRAPH OF GROWTH RATE VERSUS T_e. THESE DATA ARE VERY SIMILAR TO THE EARLIER RESULTS OF VON SENGBUSCH. THE LINEAR THEORY CURVES, BASED ON STUDIES OF EQULIBRIUM MODELS, ARE VERY FAMILIAR

GIVING BLUE EDGES OF THE INSTABILITY STRIP FOR THE FIRST TWO RADIAL MODES AS TABULATED BY MANY SUCH AS IBEN AND COX, KING, AND TABOR. THE NONLINEAR SOLUTION STABILITY IS GIVEN FOR THE OVERTONE GROWING OUT OF THE FUNDAMENTAL AND THE FUNDAMENTAL GROWING OUT OF THE OVERTONE. IN A SMALL REGION EITHER MODE CAN EXIST AT FULL AMPLITUDE. EXCEPT AT THE RED REGION BEYOND THE DEUPREE RED EDGE AT 6350 K, THE FULL AMPLITUDE SOLUTIONS ARE NEVER SIMULTANEOUSLY UNSTABLE AGAINST A SWITCH TO EACH OTHER.

THE LINEAR THEORY GROWTH RATES FOR OUR AI VEL CASE IS ON FIGURE 2, AND OUR NONLINEAR MODEL IS AT LOG T_E OF 3.875.

WITH THE GROWTH RATE OF ONLY A FEW PARTS IN A MILLION EACH PERIOD, I HAVE NOT BEEN ABLE TO GET REALLY TIGHT CONVERGENCE OF THE PERIODIC STELLINGWERF METHOD. THE VELOCITY CURVE AT THE LIMITING AMPLITUDE IS GIVEN IN FIGURE 3 FOR THREE OF THE LAST TRIAL PERIODS CALCULATED. THE VELOCITY IS ABOUT ± 7 KM/S. THE LIGHT CURVE IS GIVEN IN FIGURE 4 FOR THESE SAME THREE TRIAL PERIODS WITH A M_{BOL} RANGE OF 1.28 TO 1.48. THE STABILITY OF THESE SOLUTIONS AGAINST DECAY TO THE OVERTONE IS - 0.1% PER FUNDAMENTAL PERIOD.

CALCULATIONS FOR THE OVERTONE SHOW THAT IT IS ALSO STABLE REJECTING THE FUNDAMENTAL MODE AT A RATE OF 0.1% PER OVERTONE PERIOD. THUS AT 7500 K ONE CAN HAVE EITHER F OR 1H BEHAVIOR.

LET ME FINISH WITH A DISCUSSION OF STATISTICS WHICH MAY BE CONTROVERSIAL. THERE WERE 9 DOUBLE-MODE DWARF CEPHEIDS IN THE FIRST TABLE. FITCH DOESN'T BELIEVE ZZ MIC SHOULD BE ON THE LIST SO THERE ARE 8. THERE ARE MORE DOUBLE-MODE STARS AMONG THE δ SCUTI VARIABLES, BUT THEY ARE DIFFICULT TO DETECT AND THEN THEY OFTEN DISPLAY NONRADIAL MODES LIKE 1 MON. IN FITCH'S LIST AT THIS CONFERENCE THERE ARE 9 OF THESE. THE NUMERATOR

IN MY FRACTION IS LIKE 20. THE DENOMINATOR IS THE NUMBER OF ALL THE δ SCUTI VARIABLES. BREGER'S LATEST REVIEW LAST SUMMER AT THE ASP MEETING HAS ONLY 130 VARIABLES IN HIS TABLE, BUT ACTUALLY 1/3 OF ALL STARS BETWEEN A2 V AND F0 V VARY. THUS DENOMINATOR SHOULD PERHAPS BE THOUSANDS. THE FRACTION OF DOUBLE-MODE δ SCUTI VARIABLES IN TWO MODES IS PROBABLY LESS THAN 10^{-2}, AND THEREFORE THE CASE IS MUCH DIFFERENT THAN THAT FOR THE MORE LUMINOUS CLASSICAL DOUBLE-MODE CEPHEIDS WITH ONE-QUARTER OF ALL WITH TWO MODES.

SWTICHING TIMESCALES FOR A NON-EVOLVING AI VEL ARE LIKELY TO BE A MILLION PERIODS OR $10^6 \times 10^4$ OR 10^{10} SECONDS. THAT IS ABOUT A THOUSAND YEARS. EVOLUTION TIMES ACROSS THE INSTABILITY STRIP ARE 10^7 YEARS. WHILE THE NONLINEAR DATA FOR MODE SWITCHING RATES VERSUS T_E, LIKE THE RR LYRAE DATA SHOWN, ARE NOT YET KNOWN FOR THE δ SCUTI VARIABLES, IT APPEARS THAT THE BEST GUESS FOR THE FRACTION OF δ SCUTI VARIABLES MODE SWITCHING MAY BE 10^{-4} POSSIBLY AS OBSERVATIONS INDICATE.

MORE WORK TO UNDERSTAND THE DOUBLE-MODE BEHAVIOR OF AI VEL AND ITS ILK IS NEEDED. MY BEST GUESS IS THAT THEY ARE ALL MODE SWITCHING AT TRANSITION LINES WHICH, DUE TO THEIR DIFFERING SURFACE HELIUM CONTENT, OCCUR ALL ACROSS THE INSTABILITY STRIP.

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TABLE 1

THEORETICAL MASSES, RADII, AND LUMINOSITIES

FOR DOUBLE-MODE DWARF CEPHEIDS

Variable	Π_0(d)	Π_1/Π_0	T_e(K)	$M_T/M_\odot$	$R_T/R_\odot$	$L_T/L_\odot$	Q_0(d)
SX Phe*	0.05496	0.778	7850	1.1 ± 0.1	1.3 ± 0.1	5 ± 1	0.0325
CY Aqr**	.06104	.744:	7930	1.4 ± 0.1	1.7 ± 0.1	10 ± 1	.0326
ZZ Mic	.0654	.763	7500	1.4 ± 0.1	1.5 ± 0.1	10 ± 1	.0327
AE U Ma	.08602	.773	7500	1.6 ± 0.1	2.2 ± 0.3	14 ± 4	.0328
RV Ari	.09313	.773	7500	1.6 ± 0.1	2.3 ± 0.3	14 ± 4	.0328
BP Peg	.10954	.772	7500	1.8 ± 0.1	3.0 ± 0.3	25 ± 5	.0328
AI Vel	.11157	.773	7620	1.8 ± 0.1	2.9 ± 0.3	25 ± 5	.0328
V703 Sco	.14996	.768	7000	1.9 ± 0.1	3.9 ± 0.5	33 ± 8	.0329
VX Hya	0.22339	0.773	6980	2.2 ± 0.1	4.8 ± 0.4	48 ± 8	0.0330

* 1st crossing assumed

** 1st or third crossing

T_e values for SX Phe, CY Aqr, AI Vel, and VX Hya from McNamara and Feltz (1978). For V703 Sco, T_e is from Jones (1975). For all variables the deep interior composition is $Z = 0.01$ with Y between .2 and .3 except for SX Phe with $Z = 0.001$ with interior Y between .2 and .3.

TABLE 2

DOUBLE-MODE DWARF CEPHEID PERIOD RATIOS

$\Pi_0 = 0^d.11$ $\qquad$ $7000 \leq T_e \leq 8000$ K

$M/M_\odot$	Compositions (Y_S, Z)					
	C 312	C-D VI	D IV	D V	D IV I*	C II
	(.25, .02)	(.28, .02)	(.29, .01)	(.24, .01)	(.10, .01)	(.195, .005)
1.5	0.768	0.765	0.770	0.770	0.773	0.773
2.0	.774	.764	.770	.773	.769	.771
2.5	0.786	0.768	0.776	0.776	0.774	0.780

* Y_S down to 250,000 K

FIGURE 1

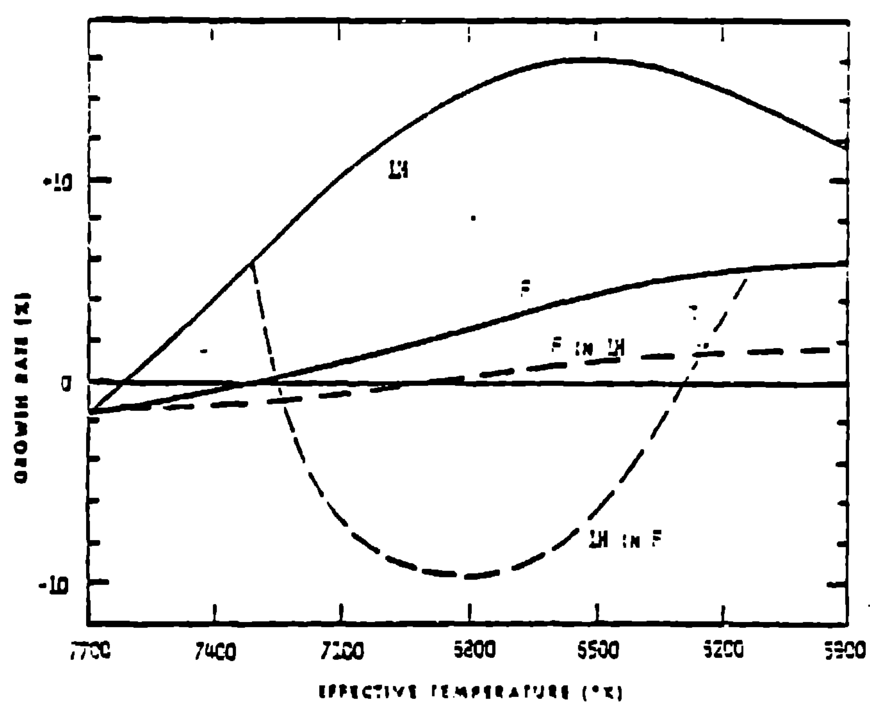

Figure 1. Composite growth rate diagram for a sequence of RR Lyrae models with $M = 0.578\ M_{\odot}$, $X = 0.7$, $Z = 0.001$, and $L = 2 \times 10^{35}$ erg/s.

FIGURE 2

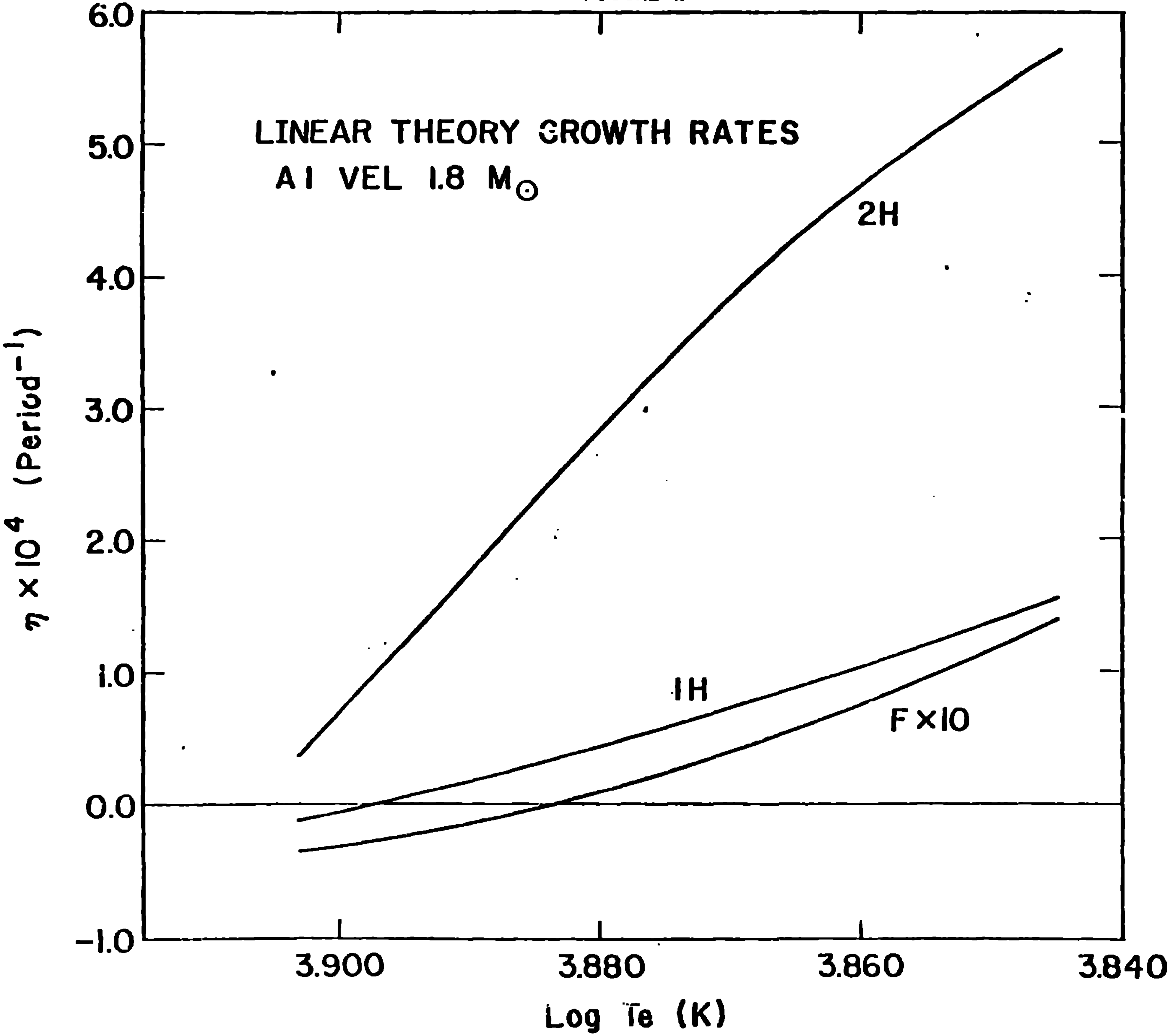
LINEAR THEORY GROWTH RATES
A I VEL 1.8 M⊙
2H
1H
F ×10
6.0
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
3.900
3.880
3.860
3.840
Log Te (K)
η × 10⁴ (Period⁻¹)

Figure 3

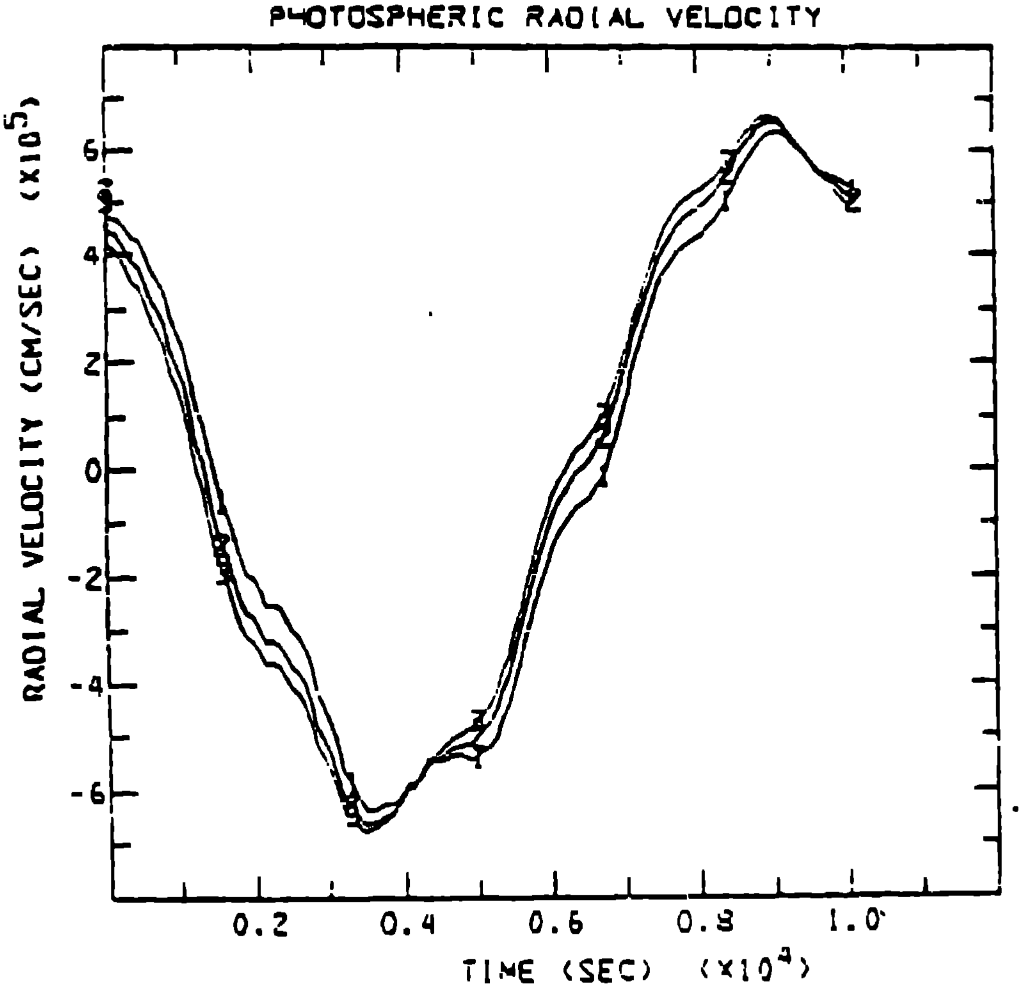
PHOTOSPHERIC RADIAL VELOCITY
RADIAL VELOCITY (CM/SEC) (X10^5)
6
4
2
0
-2
-4
-6
0.2
0.4
0.6
0.8
1.0
TIME (SEC) (X10^4)

Figure 4